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Engineering Notes

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Impact Angle Constrained Interception of Stationary Targets

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I. Introduction

IN MANY advanced guidance applications [1–5], it is required to intercept the target from a particular direction, that is, achieve a certain impact angle.

Lu et al. [6] have solved the problem of guiding a hypersonic gliding vehicle in the terminal phase to a stationary target using adaptive guidance. In their law, the missile applies maximum lateral acceleration in a sense opposite to the sense of rotation of line of sight and orients itself in a feasible geometry for the proportional navigation guidance to achieve the desired impact angle. In surface-to-surface engagements with high heading errors, such an approach is not feasible, as applying maximum lateral acceleration in the initial phase of the missile flight will cause immense induced drag. Secondly, due to rotation of the missile velocity vector in a sense opposite to the rotation of line of sight will drive the missile away from the collision course and also increase the time of flight.

In the present paper, a proportional navigation-based guidance law is proposed for capturing all possible impact angles in a surface-to-surface planar engagement against a stationary target. The achievable set of impact angles is derived for proportional navigation guidance law with $N \geq 2$. To achieve the remaining impact angles, an orientation guidance scheme is proposed for the initial phase of the missile trajectory. The orientation guidance law is also proportional navigation with the navigation constant being $N < 2$ and is a function of the initial engagement geometry and the desired impact angle. After following the orientation trajectory, the missile can switch over to a navigation constant $N \geq 2$ to achieve the desired impact angle. It is to be noted that, varying the value of the navigation constant, the proportional navigation guidance law gives a set of impact angles against stationary targets. However, studies on classical proportional navigation guidance [7] reveal that, for $N < 2$, the missile lateral acceleration shoots to infinity as the missile-target range goes to zero. But, by using $N \geq 2$ near interception in the proposed guidance law, we avoid the lateral acceleration command to shoot up to infinity and still achieve the desired impact angle.

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II. Impact Angles Against a Stationary Target with Proportional Navigation

Consider the planar engagement scenario as shown in Fig. 1a. The target is stationary and the guidance objective is to intercept the target along a desired impact angle denoted as α_{mf}. Here α_m and θ are the missile heading and the line-of-sight angle, respectively.

Proportional navigation (PN) guidance law is defined as

$$\dot{\alpha_m} = N\dot{\theta} \tag{1}$$

Integrating Eq. (1), we get

$$\frac{\alpha_{mf} - \alpha_{m0}}{\theta_f - \theta_0} = N \tag{2}$$

For the successful interception of a stationary target, the final missile heading should point toward the target, that is,

$$\theta_f = \alpha_{mf} \tag{3}$$

Using Eqs. (2) and (3),

$$N = \frac{\alpha_{mf} - \alpha_{m0}}{\alpha_{mf} - \theta_0} \tag{4}$$

Solving for the impact angle α_{mf},

$$\alpha_{mf} = \frac{\theta_0}{(1 - 1/N)} - \frac{\alpha_{m0}}{(N - 1)} \tag{5}$$

Equation (5) shows that the final impact angle is a function of the navigation constant N, other parameters being constant for a given initial engagement geometry.

The limiting impact angles using PN are

$$\alpha_{mf} = \begin{cases} 2\theta_0 - \alpha_{m0} & \text{if } N = 2 \\ \theta_0 & \text{if } N \to \infty \end{cases} \tag{6}$$

that is,

$$\alpha_{mf} \in [2\theta_0 - \alpha_{m0}, \theta_0], \qquad N \geq 2 \tag{7}$$

The achievable impact angles using PN guidance for a surface-to-surface engagement (with $\theta_0 = 0$) lie in the shaded region, as shown in Fig. 1b. Impact angles with $N < 2$ satisfying Eq. (4) cannot be achieved by PN guidance because the lateral acceleration demand goes to infinity near the interception [7].

III. Orientation Guidance

In a surface-to-surface engagement, the desired set of impact angles should contain all angles from 0 to $-\pi$. As shown earlier, classical PN ($N \geq 2$) guidance will not cover the desired range of impact angles completely. For all impact angles outside the range given by Eq. (7), we propose an orientation guidance for the initial phase of the missile flight. The missile follows the orientation trajectory as shown in Fig. 1c, until the value of N satisfying the following relation becomes equal to two:

$$\frac{\alpha_{mf} - \alpha_m}{\theta_f - \theta} = N \tag{8}$$

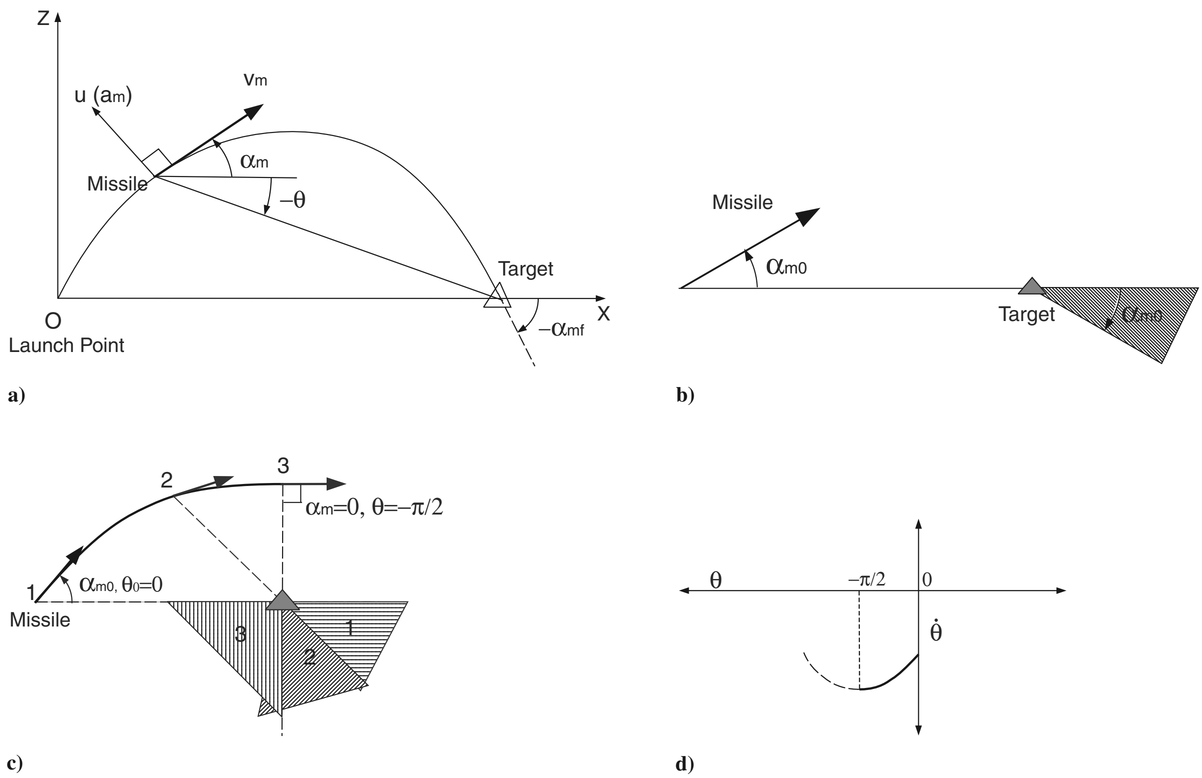


Fig. 1 Engagement scenarios: a) engagement geometry, b) PN impact angle zone, c) orientation trajectory, d) line-of-sight rate variation on orientation trajectory.

after which, the missile follows PN guidance with $N = 2$. As shown in Fig. 1c, the achievable impact angle band using PN guidance at the time of firing the missile is the shaded region 1. As the missile reaches point 2 on the orientation trajectory, the achievable band shifts to the shaded region 2. The purpose of the orientation guidance is to eventually take the missile to point 3 in Fig. 1c. At point 3, $\theta = -\pi/2$ and $\alpha_m = 0$, and thus, using Eq. (7), we find that the impact band covers $\alpha_{mf} = -\pi/2$ to $\alpha_{mf} = -\pi$ as shown by the shaded region 3. The union of all shaded impact angle regions formed by tracing the orientation trajectory is $\alpha_{mf} \in [0, -\pi]$. The properties of orientation guidance are obtained analytically in the next subsection.

A. Orientation Guidance Command

For orientation guidance, we propose the guidance law

$$a_m = N v_m \dot{\theta} \tag{9}$$

To execute the orientation maneuver, that is, to take the missile from $\theta = 0$ and $\alpha_m = \alpha_{m0}$ to $\theta = -\pi/2$ and $\alpha_m = 0$, we choose the navigation constant as

$$N = \frac{\alpha_{m0} - 0}{0 - (-\pi/2)} = \frac{2}{\pi}\alpha_{m0} \tag{10}$$

Note that

$$N \in (0, 2) \quad \forall\ \alpha_{m0} \in (0, \pi) \tag{11}$$

From Eqs. (9) and (10), the orientation guidance command is given by

$$a_m = \frac{2}{\pi}\alpha_{m0} v_m \dot{\theta} \tag{12}$$

B. Properties of the Orientation Trajectory

Using Eq. (12) we have, on the orientation trajectory,

$$\dot{\alpha_m} = \frac{a_m}{v_m} = \frac{2}{\pi}\alpha_{m0}\dot{\theta} \tag{13}$$

Integrating with respect to time,

$$\alpha_m = \frac{2}{\pi}\alpha_{m0}\theta + \alpha_{m0} \tag{14}$$

Equation (14) relates the missile heading and the line-of-sight angle on the orientation trajectory.

Proposition 1: On the orientation trajectory, the line-of-sight rate $\dot{\theta} < 0$ and the missile velocity vector rotation rate $\dot{\alpha}_m < 0$.

Proof: For a stationary target

$$\dot{\theta} = -\frac{v_m}{R}\sin(\alpha_m - \theta) \tag{15}$$

Using Eq. (14) in Eq. (15),

$$\dot{\theta} = -\frac{v_m}{R}\sin\left[\alpha_{m0} + \left(\frac{2}{\pi}\alpha_{m0} - 1\right)\theta\right] \tag{16}$$

$$\Rightarrow \dot{\theta} < 0, \quad \text{for all } \theta \in [0 - \pi/2], \alpha_{m0} \in (0, \pi) \tag{17}$$

with

$$\dot{\theta} = \begin{cases} -\frac{v_m}{R}\sin\alpha_{m0} & \text{if } \theta = 0 \\ -\frac{v_m}{R} & \text{if } \theta = -\frac{\pi}{2} \end{cases} \tag{18}$$

The line-of-sight rate variation with respect to line-of-sight angle is shown with a solid line in Fig. 1d. Also, using Eq. (13),

$$\dot{\alpha_m} = \frac{2}{\pi}\alpha_{m0}\dot{\theta} \tag{19}$$

From Eqs. (17) and (19),

$$\dot{\alpha_m} < 0, \quad \forall\ \theta \in [0 \quad -\pi/2], \quad \alpha_{m0} \in (0, \pi) \tag{20}$$

□

Proposition 2: On the orientation trajectory, $\bigcup_{\theta\in[-\pi/2 \quad 0]}[2\theta - \alpha_m \quad \theta] = [-\pi \quad 0]$.

Proof: Let

$$q_1 = 2\theta - \alpha_m \tag{21}$$

$$q_2 = \theta \tag{22}$$

Differentiating q_1 with respect to time,

$$\dot{q}_1 = \dot{\theta}\left(2 - \frac{\dot{\alpha_m}}{\dot{\theta}}\right) \tag{23}$$

Using Eq. (13) in Eq. (23),

$$\dot{q}_1 = \dot{\theta}\left(2 - \frac{2}{\pi}\alpha_{m0}\right) \tag{24}$$

Since $\dot{\theta} < 0$ (Proposition 1) and $\alpha_{mo} \in (0, \pi)$,

$$\dot{q}_1 < 0 \tag{25}$$

Similarly, for q_2, using Eq. (22) and Proposition 1,

$$\dot{q}_2 = \dot{\theta} < 0 \tag{26}$$

Let q_{1i} and q_{2i} be the values of q_1 and q_2 at the initial point (see point 1 on the missile trajectory in Fig. 1c) on the orientation trajectory. At the initial point, $\theta = 0$ and $\alpha_m = \alpha_{m0}$. Using Eqs. (21) and (22), we have

$$q_{1i} = -\alpha_{m0}, \qquad q_{2i} = 0 \tag{27}$$

Also, let q_{1t} and q_{2t} be the values of q_1 and q_2 at the terminal point (see point 3 on the missile trajectory in Fig. 1c) on the orientation trajectory. At the terminal point, $\theta = -\pi/2$ and $\alpha_m = 0$. Using Eqs. (21) and (22), we have

$$q_{1t} = -\pi, \qquad q_{2t} = -\pi/2 \tag{28}$$

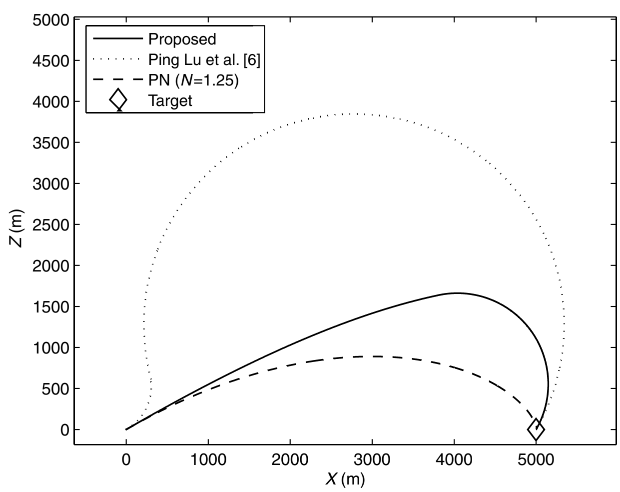
Thus,

$$\cup_{\theta\in[-\pi/2 \quad 0]}[2\theta - \alpha_m \quad \theta] = \cup_{\theta\in[-\pi/2 \quad 0]}[q_1 \quad q_2] \tag{29}$$

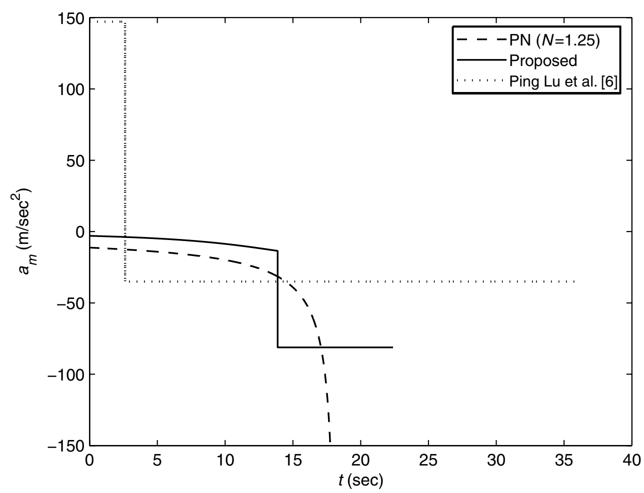
$$= [q_{1i} \quad q_{2i}] \cup [q_{2t} \quad q_{2i}] \cup [q_{1t} \quad q_{1i}] (\text{since } \dot{q}_1 < 0, \dot{q}_2 < 0) \tag{30}$$

$$= [-\alpha_{m0} \quad 0] \cup [-\pi/2 \quad 0] \cup [-\pi \quad -\alpha_{m0}] = [-\pi \quad 0] \tag{31}$$

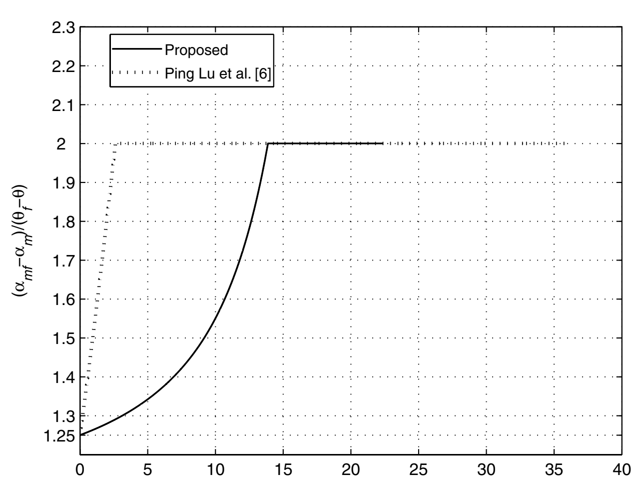
□



a) Missile trajectories



b) Lateral acceleration



c) $\frac{\alpha_{mf} - \alpha_m}{\theta_f - \theta}$ **vs time**

Fig. 2 Results, case 1.

IV. Composite Guidance Law

Proposition 2 shows that there exists a point on the orientation trajectory so that, if we consider that point as the initial missile position, then PN guidance law with $N \geq 2$ will achieve any impact angle in $[-\pi \quad 0]$. The proposed composite PN guidance law follows the orientation guidance command given by Eq. (12) if the value of N satisfying Eq. (8) is less than two, until Eq. (8) is satisfied with $N = 2$. After which, PN guidance with $N = 2$ is used. The proposed composite PN guidance law is given as

$$a_m = N v_m \dot{\theta} \tag{32}$$

For engagement geometries with

$$\frac{\alpha_{mf} - \alpha_{m0}}{\theta_f - \theta_0} \geq 2$$

$$N = \frac{\alpha_{mf} - \alpha_{m0}}{\theta_f - \theta_0} \tag{33}$$

For engagement geometries with

$$\frac{\alpha_{mf} - \alpha_{m0}}{\theta_f - \theta_0} < 2$$

$$N = \begin{cases} \frac{2}{\pi}\alpha_{m0} & \text{if } \frac{\alpha_{mf}-\alpha_m}{\alpha_{mf}-\theta} < 2 \\ 2 & \text{if } \frac{\alpha_{mf}-\alpha_m}{\alpha_{mf}-\theta} = 2 \end{cases} \tag{34}$$

For realistic simulations, a gravity compensation term is added to Eq. (32):

$$a_m = N v_m \dot{\theta} + g \cos\alpha_m \tag{35}$$

where N is given by Eqs. (33) and (34).

V. Simulations Results

To demonstrate the basic properties of the proposed guidance law, we use a constant speed missile model. To prove the applicability of the proposed guidance law in a realistic scenario, we also use a realistic point mass missile model as a point mass flying over a flat, nonrotating Earth with given aerodynamic and thrust profiles. The detailed model is borrowed from Kee et al [8].

A. Constant Speed Missile Model

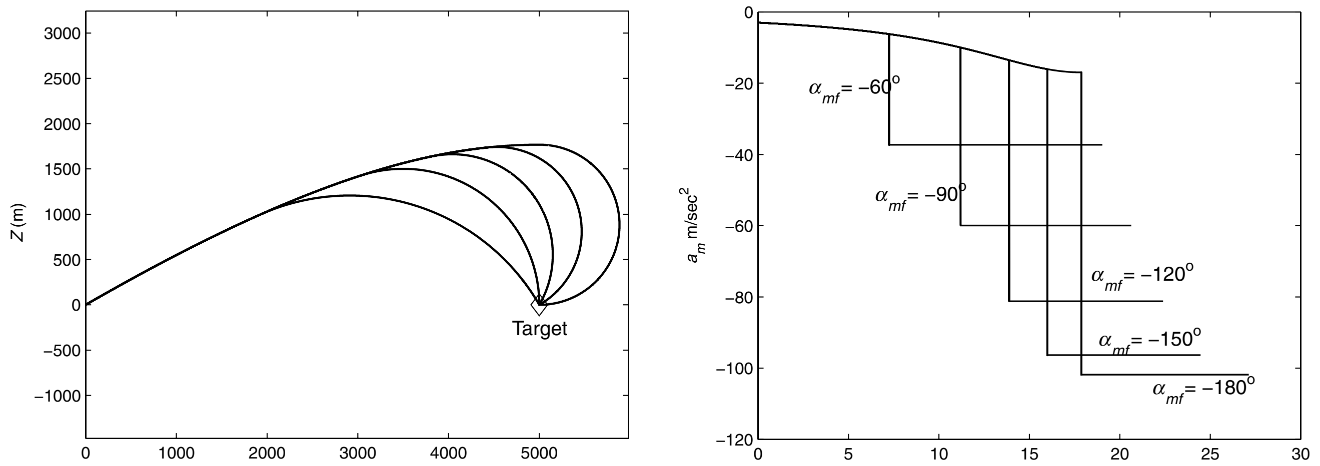
The missile speed $v_m = 300$ m/s with $(x_{m0}, z_{m0}) = (0, 0)$ and $(x_{t0}, z_{t0}) = (5000$ m$, 0)$. Simulations are terminated for $R < 0.1$ m, and the corresponding impact angle errors are less than 10^{-5} deg.

1. *Case 1:* $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) < 2$

In this case, we consider engagement geometries with $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) < 2$. We choose $\alpha_{m0} = 30$ deg and $\alpha_{mf} = -120$ deg for the simulation. The corresponding value of $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) = 1.25$. Solid lines in Figs. 2a and 2b show the missile trajectory and lateral acceleration history, respectively. The missile follows the orientation trajectory in the first phase guidance and switches to PN guidance with $N = 2$ as the value of the expression $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0)$ increases to two, as shown in Fig. 2c. Results for classical PN with $N = 1.25$ are shown in Fig. 2 with dashed lines. As expected, the lateral acceleration saturates to the value of $-15g$ near the end, resulting in a miss distance of 13.05 m with an impact angle error of 45.99 deg. The results show that the proposed guidance law achieves impact angles which cannot be achieved by classical PN guidance. With the same engagement geometry, a comparative study is carried out with the proportional navigation-based composite impact angle constrained guidance law by Lu et al. [6]. Results are plotted in Fig. 2 with dotted lines. Figure 2a shows the comparison of the missile trajectories of the two guidance laws. Missile trajectory guided by the existing guidance law applies the maximum possible lateral acceleration of $15g$ (see Fig. 2b) to rapidly increase the value of the expression $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0)$ to two (see Fig. 2c). By executing such a maneuver, the missile is forced away from the collision course to orient itself to follow PN guidance for the desired impact angle. The proposed PN orientation guidance scheme, on the other hand, applies a guidance command which is of the same sign as the line-of-sight rate. This keeps the missile closer to the collision course and requires less control effort with less time of flight. The total control effort $\int a_m^2 \, dt$ for the proposed guidance law is 56,829 units, as compared with Lu et al.'s guidance law, which uses 97,994 units.

2. *Case 2:* $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) < 2$ *with Different Impact Angles*

The engagement parameters for this case are same as in case 2 with impact angles $\alpha_{mf} = -60, -90, -120, -150$, and -180 deg. Missile trajectories and lateral acceleration profiles for the case are plotted in Figs. 3a and 3b, respectively. The results prove the capability of the proposed guidance law for capturing all impact angles outside the capturability of the classical PN guidance.



a) Missile trajectories **b) Lateral acceleration**

Fig. 3 Results, case 2.

B. Realistic Missile Model

In this subsection, simulations are carried out with the realistic missile model with $(x_{m0}, z_{m0}) = (0, 0)$ and $(x_{t0}, z_{t0}) = (10,000\ \mathrm{m}, 0)$. All simulations are terminated for $R < 0.1$ m, and the corresponding impact angle errors are less than 10^{-3} deg. The model is taken from Kee et al [8].

1. *Case 3:* $(\alpha_{mf} - \alpha_{m0})/(\theta_f - \theta_0) < 2$

In this study, we consider desired impact angles to be outside the capture region of the classical PN guidance law. The missile is fired with $\alpha_{m0} = 30$ deg with desired impact angles $\alpha_{mf} = -60, -90, -120, -150$, and -180 deg. As the guidance loop is closed at $t = 1.5$ s and $\theta(t = 1.5) \neq 0$, the orientation command derived from Eq. (10) is modified as follows:

$$N = \frac{(-0 - \alpha_{\mathrm{mglc}})}{[-(\pi/2) - \theta_{\mathrm{glc}}]} = \frac{\alpha_{\mathrm{mglc}}}{[(\pi/2) + \theta_{\mathrm{glc}}]} \tag{36}$$

where α_{mglc} and θ_{glc} are the missile heading and line-of-sight angle, respectively, at the time of guidance loop closure. The trajectories are plotted in Fig. 4a, which shows the successful interception of the target. The missile follows the orientation command first (see Fig. 4b) and then switches to PN guidance with $N = 2$. The lateral acceleration demand is higher for trajectories with higher overall angular turn. The corresponding speed profiles are plotted in Fig. 4c. Trajectories with higher lateral acceleration demand have higher induced drag, resulting in greater loss of speed. The case study shows that the proposed guidance law achieves all impact angles outside the capture region of the classical PN guidance in a realistic surface-to-surface engagement scenario.

2. *Case 4: Missile with First-Order Autopilot Lag*

The analysis carried out in this paper is based on Eq. (1). Realistic missile systems have autopilot lag while executing the guidance command, and Eq. (1) is no longer valid. The proposed guidance law, in such cases, can be applied in a feedback form as follows:

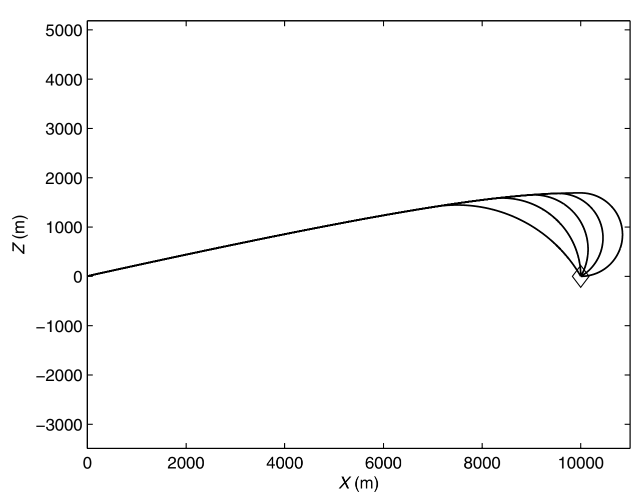
Orientation navigation constant

$$N = \frac{(-0 - \alpha_m)}{[-(\pi/2) - \theta]} = \frac{\alpha_m}{[(\pi/2) + \theta]} \tag{37}$$

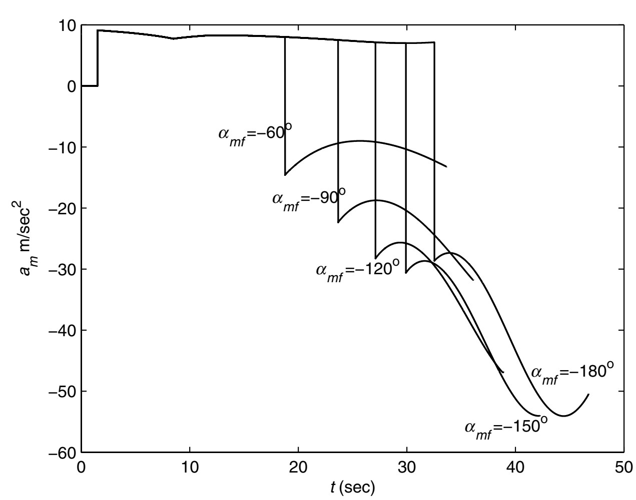
Terminal navigation constant

$$N = \frac{(\alpha_{mf} - \alpha_m)}{(\alpha_{mf} - \theta)} \tag{38}$$

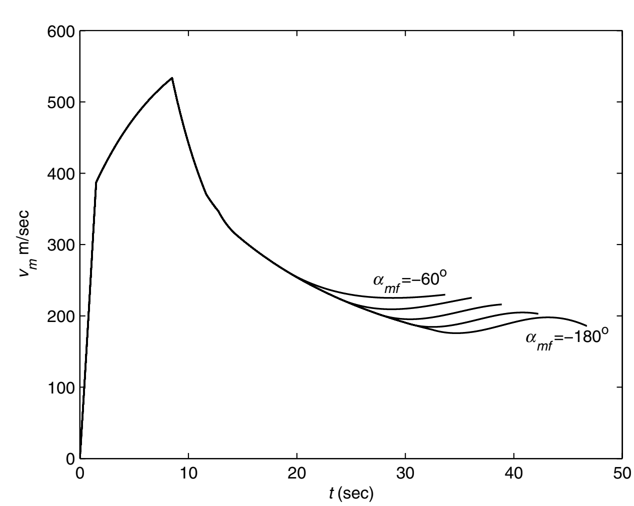
Note that the navigation constants given by Eqs. (37) and (38) are no longer constants and are updated every guidance cycle. For simulations, we consider a first-order autopilot lag with the time constant τ. Missile lateral acceleration profiles for $\alpha_{m0} = 60$ deg, $\alpha_{mf} = -120$ deg, and $\tau = 0.2$ s are plotted in Fig. 5a. The error in impact angle is 0.0488 deg. Impact angle error vs τ for different impact angles is plotted in Fig. 5b. The impact angle errors are less than 0.2 deg for uncompensated first-order delays up to 0.4 s. The



a) Missile trajectories

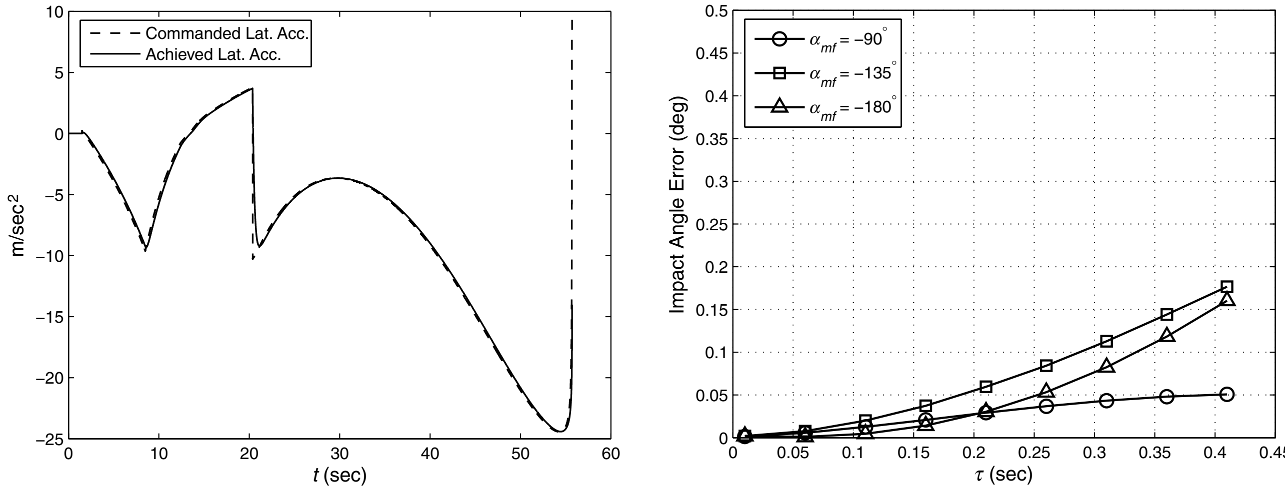


b) Lateral acceleration



c) Missile speed profiles

Fig. 4 Results, case 3.



a) Lateral acceleration **b) Impact angle error vs τ**

Fig. 5 Results, case 4.

results show that the guidance law can be implemented with autopilot lag with negligible error in any desired impact angle.

VI. Conclusions

In this paper, a composite guidance law is proposed for intercepting stationary targets with terminal impact angle constraint. The achievable impact angle set for the conventional PN guidance law with $N \geq 2$ is limited. The proposed guidance law achieves all possible impact angles in a surface-to-surface engagement scenario. To obtain the impact angles outside the achievable set of conventional PN guidance law, an orientation guidance strategy is proposed with a navigation constant $N < 2$. The orientation navigation constant depends on the initial engagement geometry and the desired impact angle. It is shown mathematically that, following the orientation trajectory, all impact angles can be achieved. Simulations are carried out for kinematic and realistic missile models separately. The guidance law is implemented in a feedback mode for missiles having autopilot lag with negligible errors in the impact angle and miss distance.

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